Reinsurance contract pricing with rare catastrophy events

Application of copulas theory

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Abstract: Reinsurance contract pricing usually involves complex terms such as franchised deductible, annual aggregate deductible, annual limit, binary payoff, and number of event triggers. Analytical approach is not suitable for this purpose and simulation method is usually used. But pricing will not only need the contract loss distribution, it will also need the company whole portfolio loss for calculating the correlation and the allocated equity related to the risk of this correlation, since allocated equity is tied to the given goal of profit return on equity of a company. With rare catastrophe events, such as some earthquake or terrorism attack events, which has occurrence frequency below 1e-7 or 1e-8 per year, simulation capturing these events for a single contract may be amenable to nowadays hardware power, but simulating the whole company portfolio loss that in addition to these rare events may also have other more frequent events like wind storms, may well beyond the hardware limits. For the correlation calculating, we proposed a mixed analytical and simulation approach by simulate the marginal distribution and the copulas between the portfolio and contracts loss. Criteria for good marginal distribution and discrete copulas simulation are studied for pricing purpose.
Keywords: rare events simulation;copulas;pricing;capital allocation

Ⅰ.Introduction

Our approach is use simulation when analytical method is too complex or unable to find a solution, and use analytical methods when simulation can not handle the huge files. For example, the payoff calculation will through simulation to accommodate the complex contract terms. After the simulation, the contract payoff will be summarized through parameters of conditional compound distributions; this distribution will be combined with various reference portfolios generating method through copulas that catch the correlation between the contract and the portfolio, to calculate the allocated equity depends on this correlation. The simplest reference portfolio selection is assume the event loss in the contract and in the reference portfolio is co-monotonic, in that case the copulas calculated through the reference portfolio alone is enough for infuse the contract loss to a portfolio combined from the reference portfolio and the contract payoff. The original portfolio loss quantile and the subset events (common to the contract) loss quantile can be used as a simulated sample of the copulas. More sophisticated reference portfolio an be done by divided it into various risk groups, simulating each subset so that their simulation will keep the original reference portfolio AEP. For the risk group in common of contract and reference portfolio, simulate the portfolio and the contract loss aligned to catch their correlations, using years large enough to capture as more events as wanted. Combine different risk groups, assuming that they are independent, and check the combined AEP.

Numerical experiments are used to confirm or test the selection of the distribution used, the reference creation method and assumptions used. Since AEP is the main numbers used in pricing, it will be used as the main cretiria and its calculation methods and usage will be the major tasks.

Ⅱ.Benchmark for good simulation

A.AEP Calculation by FFT

FFT method's max loss selection, how many points to calculate, tilting, and padding parameters choices are studied. A compound Poisson distribution with Uniform severity on [0,1] is used as a benchmark example, which have an analytical formula representation through modified Bessel function of the first kind. When 2^{14} or 2^{17} points are used, tilting has almost no effects on AEP loss for return periods up to 1 million years. Tilting will have effects when AEP loss is bigger than half of the chosen max loss. The best tilting parameters may be severity distribution dependent. In our Uniform distribution case, the best tilting parameter is between 25 and 26. But when tilt is used the calculated aggregate loss CDF is not increasing and it actually will fluctuate when near 1 and may have probability far bigger than 1 or have negative numbers. Empirically the effect of a tilt of 20 is close to that of a padding by 16 times the number of discretized loss values used for calculating severity CDF or PDF. The padding methods do not have the problem of non-increasing CDF or negative PDF as when from tilting.

B.AEP Calculation by Panjer recursion

When all other parameters unchanged, increasing the points calculated, or equivalently, fix the points numbers but decrease the max loss, the FFT calculated AEP loss will increase. For find the limit, Panjer recursion method was used, which when points increasing, or fixing the points but decreasing the max loss, will decrease. And the Panjer recursion methods always have AEP loss bigger than FFT AEP loss. It is also find that the relative error between Panjer recursion and FFT, when the points is doubled, will be halved. In one example, when 2^{17} points and 16 times padding is used, the relative error is 0.4%. When max loss reduced by 4 times, the relative error decreased to 0.1%. And it is found that the average of the Panjer and FFT number is a good approximation of (though slightly bigger than) the final limit.

With this limit as reference, it is found that the formula approximation calculation [1] when period is 10 million years will be 4% less than accurate value, and when 100000 years will be 5% less, for smaller period will be up to 23% less. So it should not be used for benchmark purpose.

The time used by Panjer recursion is only about a quarter to a half more than the corresponding FFT with padding([2]).

Empirical study also shows 1 to 11 weighted mean of Panjer number and FFT number will be a good approximation to the accurate AEP number.

Ⅲ.Reference Portfolio Choice

Reference portfolio have different kind of risk groups, some are using compound Poisson distribution, others used mixed NB and Poisson for frequency distribution, still others used composite discrete distributions. And some risk group used various severity distribution, while the majorities used Beta distribution. Since the reference portfolio and the contract to be priced used the same set of pre-simulated events occurrence table, events will occur or missing at the same time, we can assume as a first approximation the correlation between them is correct, regardless of whether events are not simulated or simulated number bigger than its rates. We will adjust the marginal distribution of the contract payoff loss according to a more comprehensive simulation results. This is the AEP compensation approach.

But if the simulated number of events for the pricing contract is too small, the correlation between it and the reference portfolio may not be captured correctly. We may want to find the loss correlation between the event subset used by the pricing contract and the whole reference portfolio event set from the reference portfolio losses alone, as a peer event group comparison, using it as the average correlation, or equivalently, as a sample of the copulas between the reference portfolio and the contract, for pricing the contract.

Viewing the peer risk reference portfolio another way, it can be regard as on average the portfolio-wise pricing result for the contract. If

we compare the AEP adjusted result with it, we can know whether the contract is better or bad than average.

Another reason for the AEP adjustment view is that when we consider the joint distribution of reference portfolio and the pricing contract, the marginal distribution of the reference portfolio can be regard as accurate since it consists of thousands of contracts and the good one and bad one will average out each other, together with the argument that the copulas between reference portfolio and contracts is stable with respect to simultaneous alteration in both of them, the only adjustment needed should be the marginal loss distribution of the pricing contracts. To testing this assumption, alternative seeds for simulation and increasing simulation years will be used to estimate the true values and the errors by AEP adjustment methods.

A.Distribution choice and fitting

We found a compound Poisson Beta distribution assumption will have AEP 5 to 6 percent less than the empirical AEP. Is this due to NB frequency distribution in some risk group, or due to mismatch of Beta distribution to severity, or even due to some weak correlation between different events originated from annual limit, FD, or number of event trigger? This part will address these problems.

B. AEP compensation

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C. Event set peer group reference

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D.Different seeds reference portfolio

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E.Synthetic reference portfolio

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Acknowledgment

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References

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